CPE409 Image Processing

Part 9
Morphological Image Processing

Assist. Prof. Dr. Caner ÖZCAN

In form and feature, face and limb, I grew so like my brother that folks got taking me for him and each for one another.
~Henry Sambrooke Leigh, Carols of Cockayne, The Twins

Outline

9. Morphological Image Processing
   - Preliminaries
   - Erosion and Dilation
   - Opening and Closing
   - Some Basic Morphological Algorithms

Introduction

- **Morphology**: a branch of biology that deals with the form and structure of animals and plants

- Morphological image processing is used to extract image components for representation and description of region shape, such as boundaries, skeletons, and the convex hull

Preliminaries (1)

- **Reflection**

The reflection of a set B, denoted $\hat{B}$ is defined as

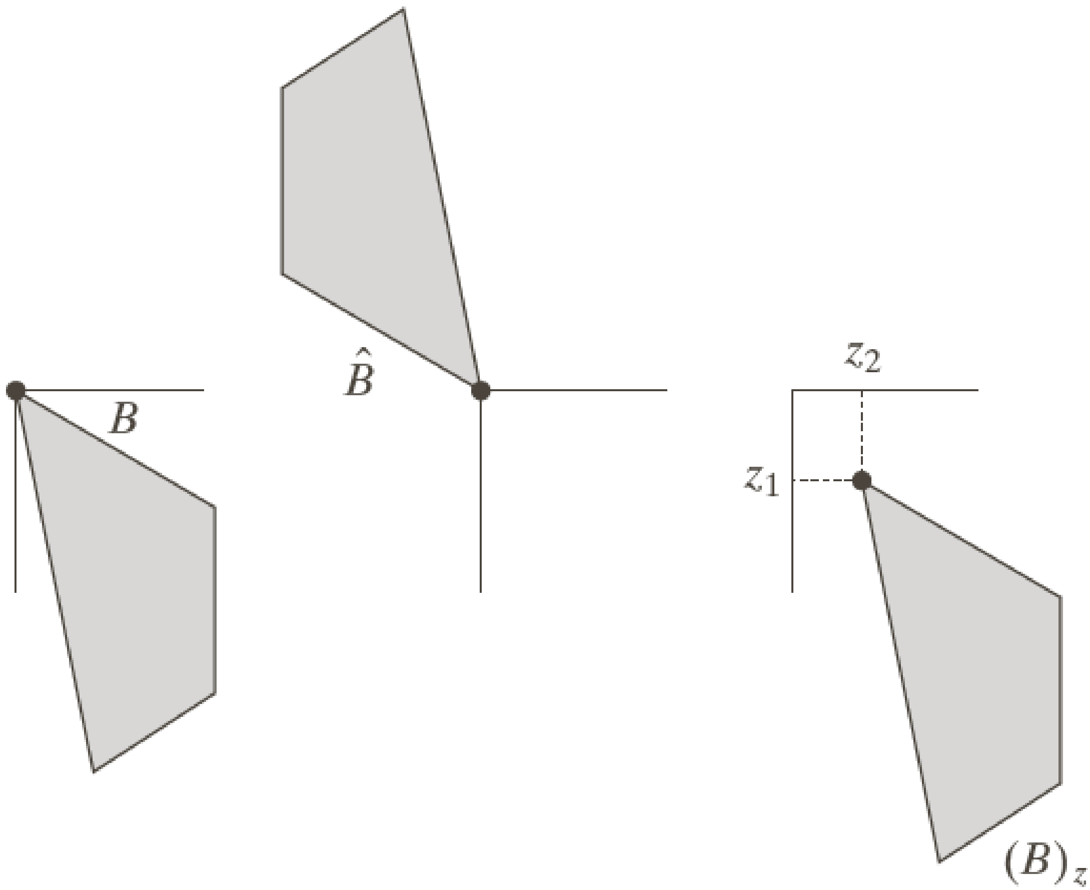
$$\hat{B} = \{w \mid w = -b, \text{for } b \in B\}$$

- **Translation**

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_Z$, is defined as

$$(B)_Z = \{c \mid c = b + z, \text{for } b \in B\}$$

Example: Reflection and Translation



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

Examples: Structuring Elements (1)

Small sets or sub-images used to probe an image under study for properties of interest

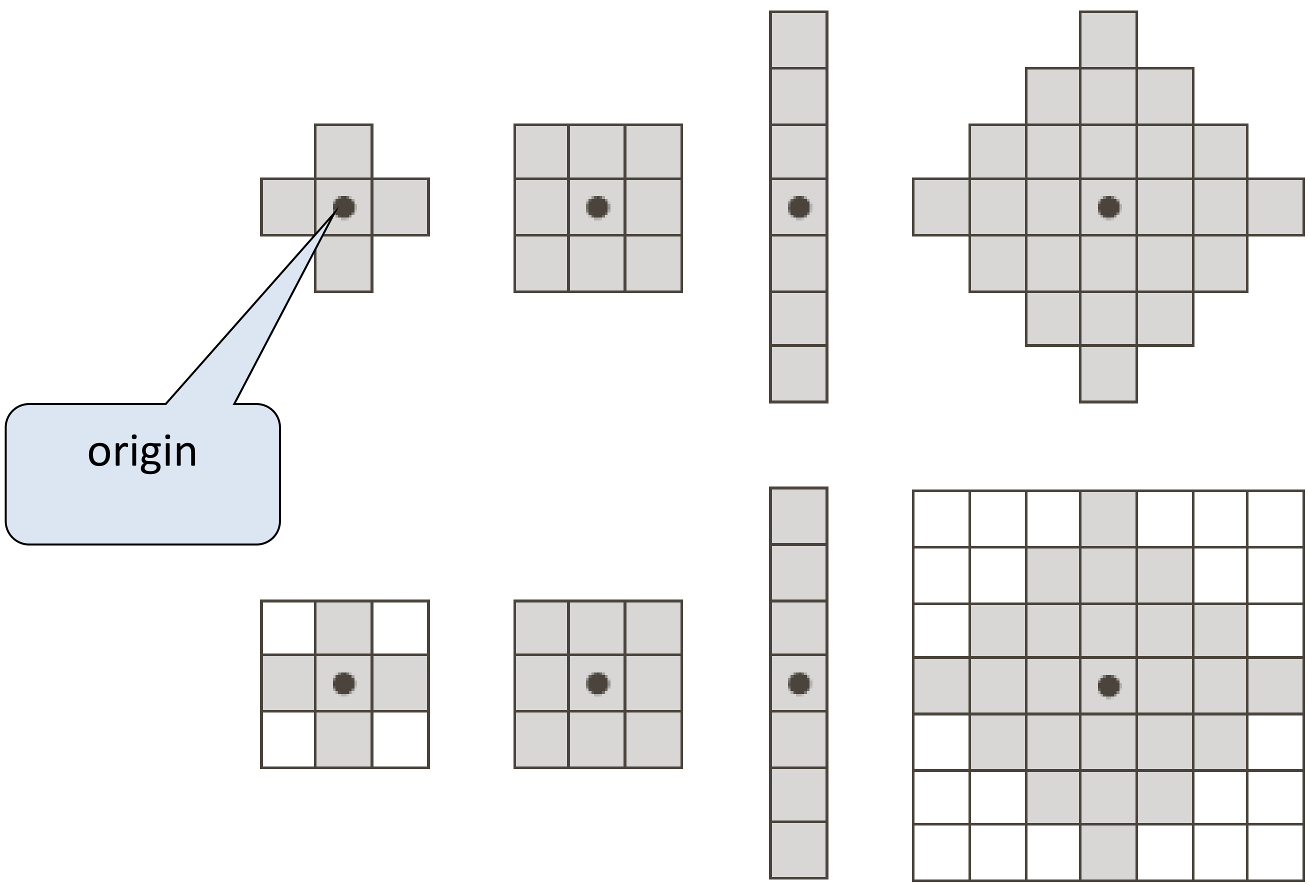


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Examples: Structuring Elements (2)

Accommodate the entire structuring elements when its origin is on the border of the original set A

Origin of B visits every element of A

At each location of the origin of B, if B is completely contained in A, then the location is a member of the new set, otherwise it is not a member of the new set.

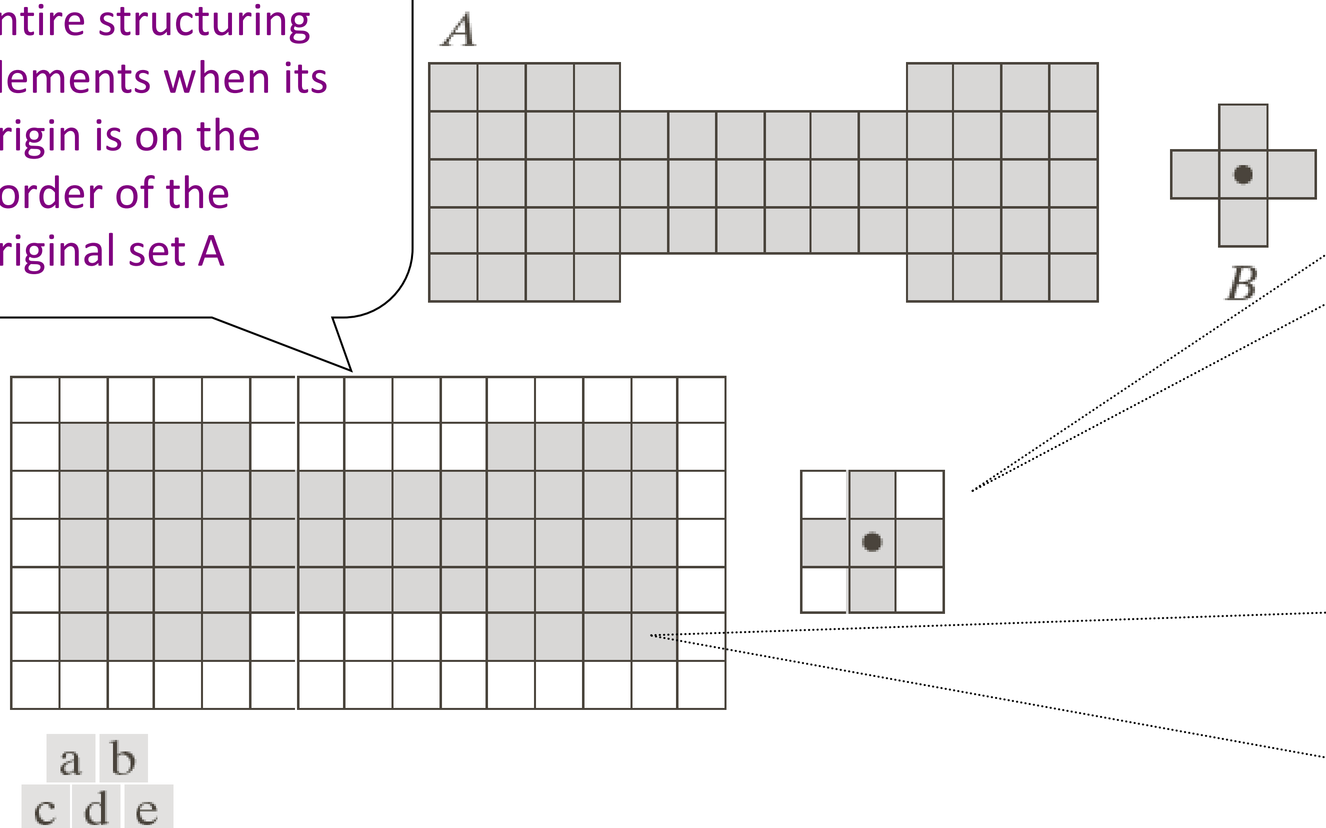


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Erosion

With A and B as sets in Z^2, the erosion of A by B, denoted $A \ominus B$ defined

$$A \ominus B = \left\{ z \mid (B)_Z \subseteq A \right\}$$

The set of all points z such that B, translated by z, is contained by A.

$$A \ominus B = \left\{ z \mid (B)_Z \cap A^c = \varnothing \right\}$$

Example of Erosion (1)

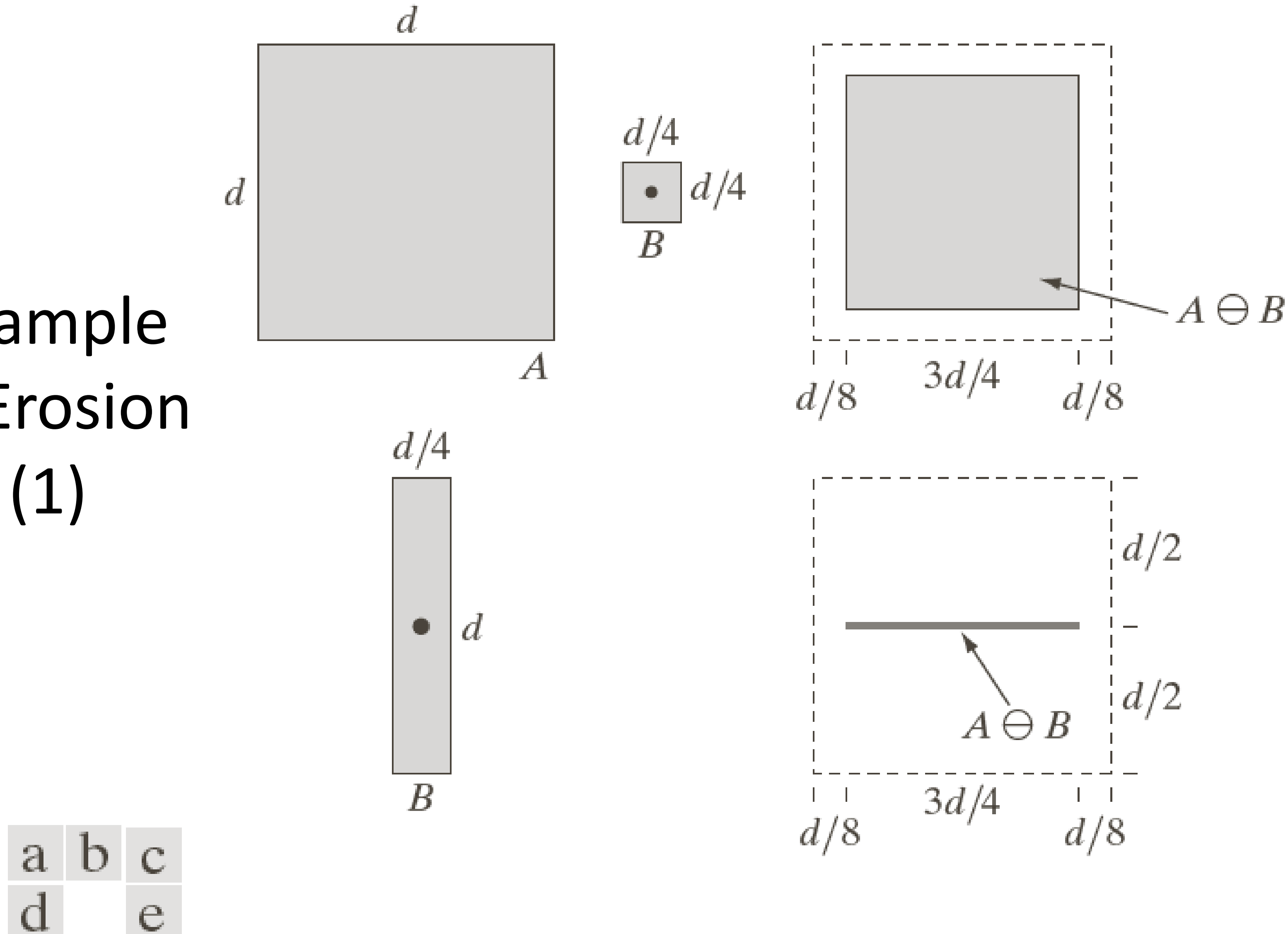


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Example of Erosion (2)

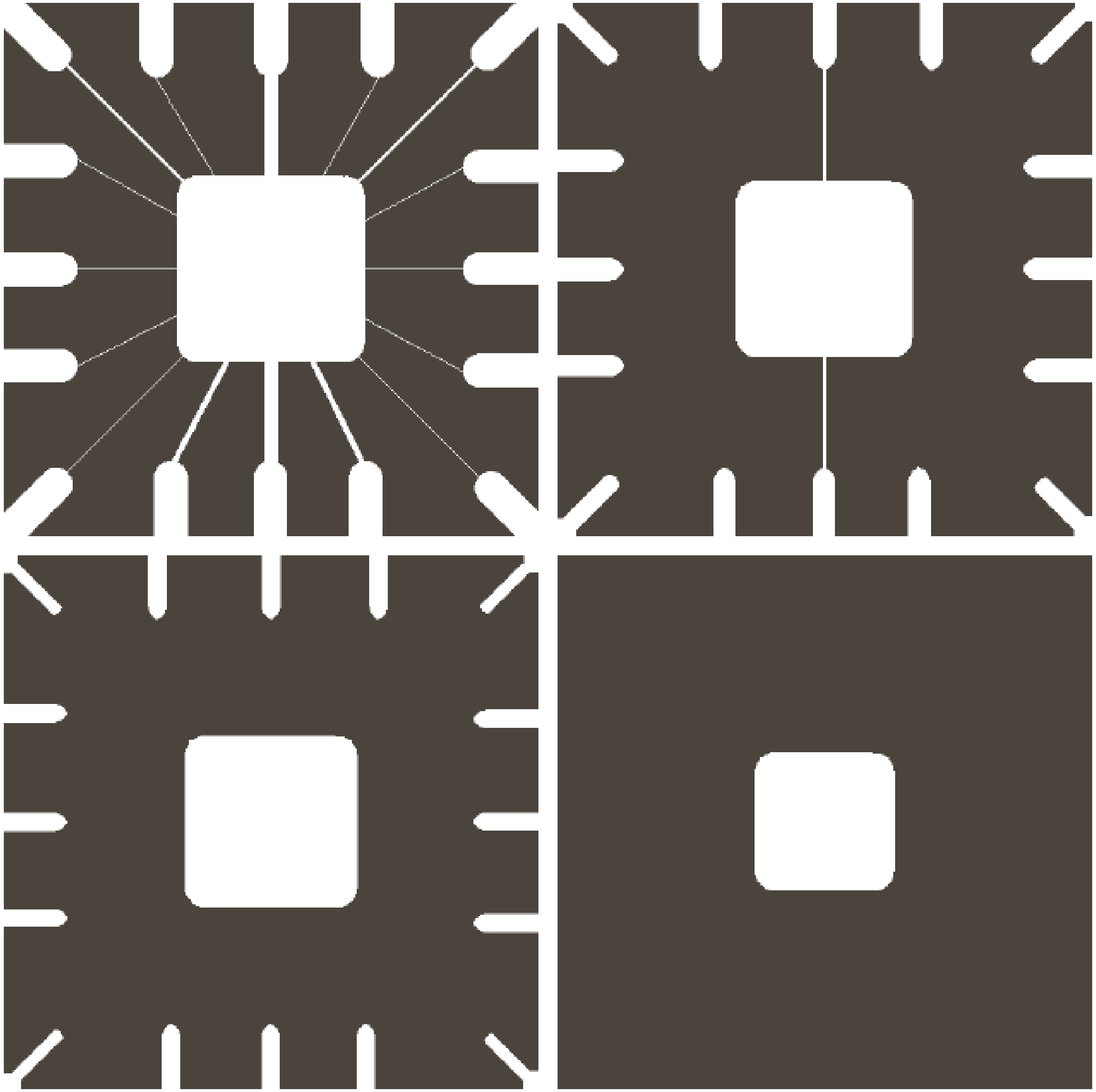


FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

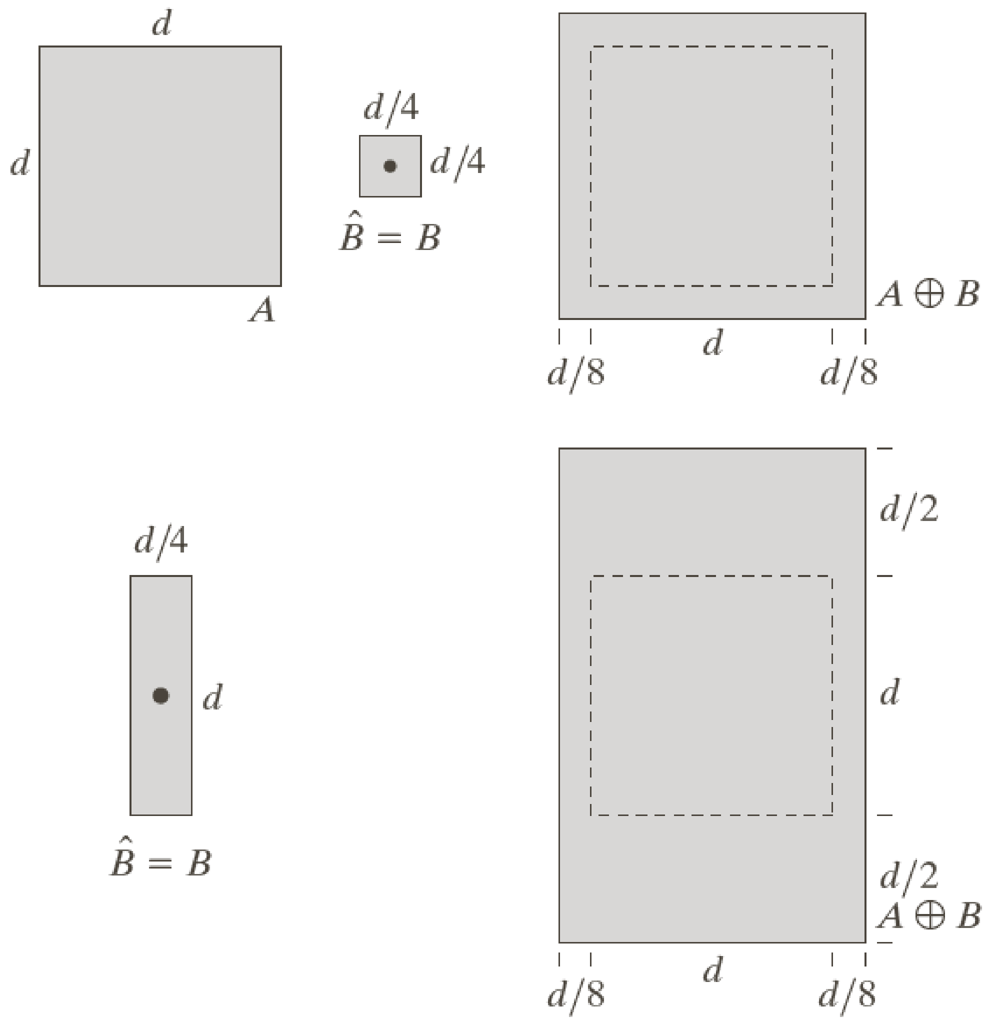
With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$\mathrm{A} \oplus \mathrm{B} = \left\{ z \mid \left(\hat{B} \right)_z \cap A \neq \varnothing \right\}$$

The set of all displacements z, the translated $\hat{B}$ and A overlap by at least one element.

$$A \oplus B = \left\{ z \mid \left[\left(\hat{B} \right)_z \cap A \right] \subseteq A \right\}$$

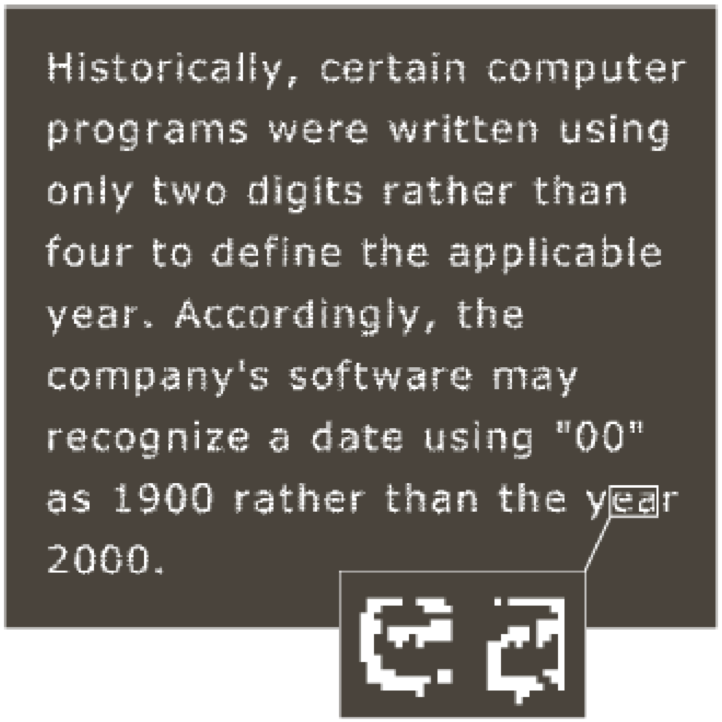
Examples of Dilation

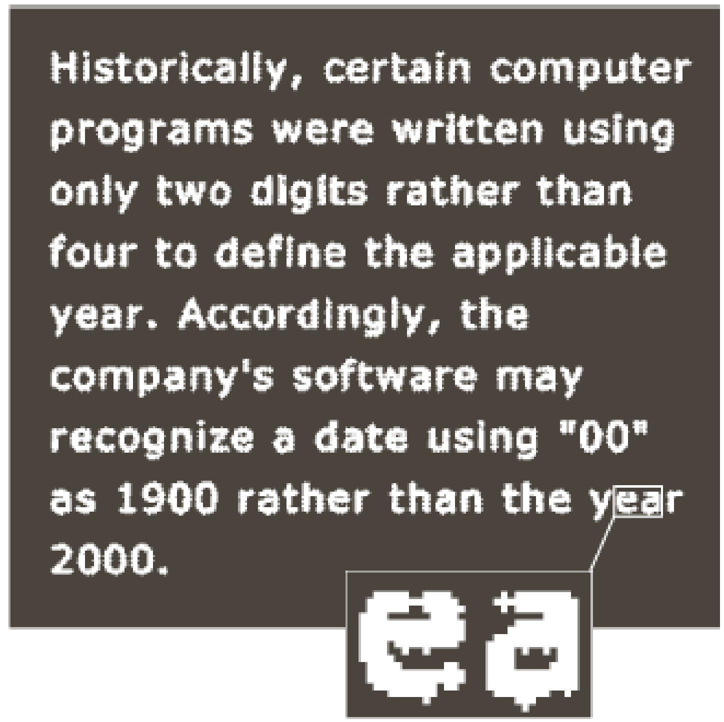


| a | b | c |
| d | | e |

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Examples of Dilation





0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$\left(A - B\right)^c = A^c \oplus \hat{B}$$

and

$$\left(A \oplus B\right)^c = A^c - \hat{B}$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$\left(A \ominus B\right)^c = \left\{z \mid \left(B\right)_Z \subseteq A\right\}^c$$

$$= \left\{z \mid \left(B\right)_Z \cap A^c = \varnothing\right\}^c$$

$$= \left\{z \mid \left(B\right)_Z \cap A^c \neq \varnothing\right\}$$

$$= A^c \oplus \hat{B}$$

Duality

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$
\begin{aligned}
(A \oplus B)^c &= \left\{ z \mid \left(\hat{B} \right)_Z \cap A \neq \varnothing \right\}^c \\
&= \left\{ z \mid \left(\hat{B} \right)_Z \cap A^c = \varnothing \right\} \\
&= A^c \ominus \hat{B}
\end{aligned}
$$

Opening and Closing

- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing tends to smooth sections of contours but it generates fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

Opening and Closing

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = \left(A \ominus B \right) \oplus B$$

The closing of set A by structuring element B, denoted $A \bullet B$ is defined as

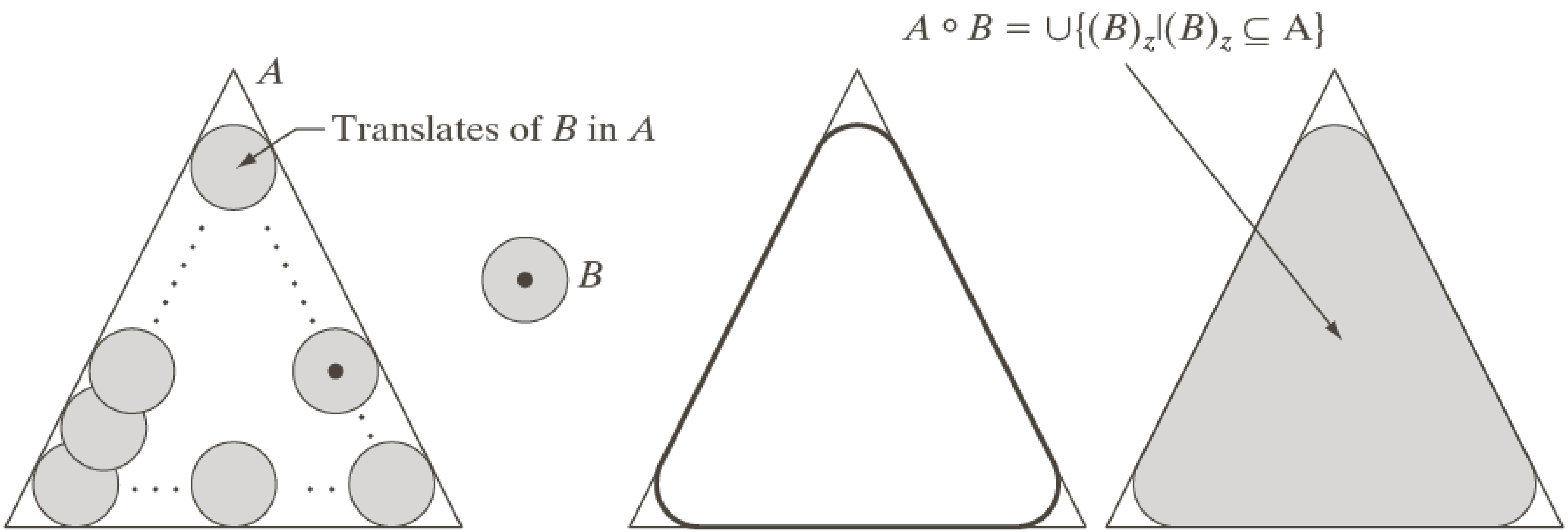
$$A \bullet B = \left(A \oplus B \right) - B$$

Opening

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = \bigcup \left\{ \left(B \right)_Z \mid \left(B \right)_Z \subseteq A \right\}$$

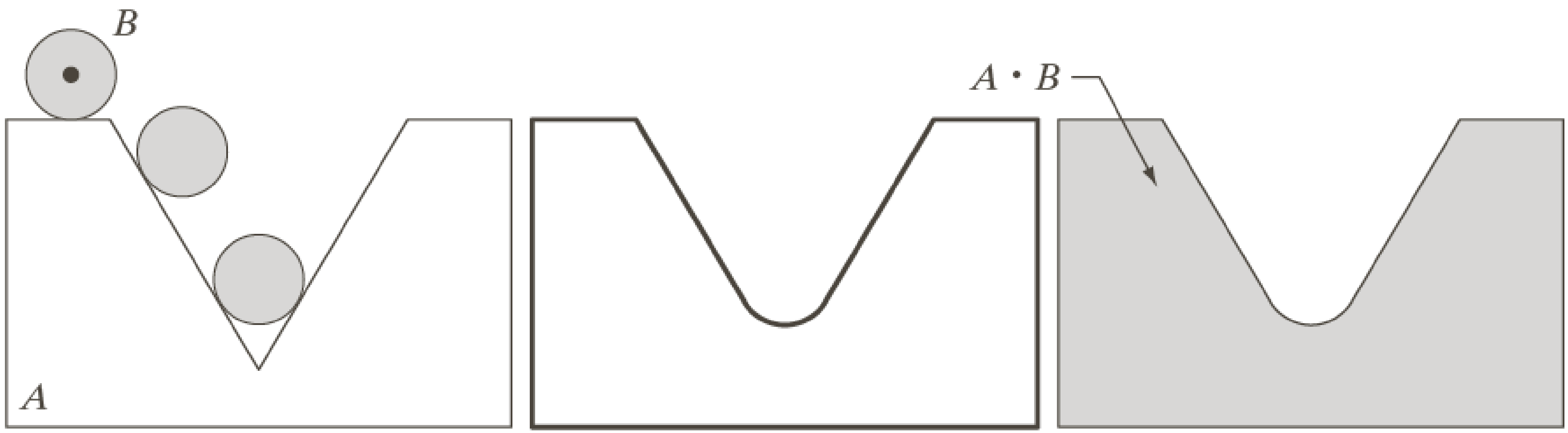
Example: Opening



a b c d

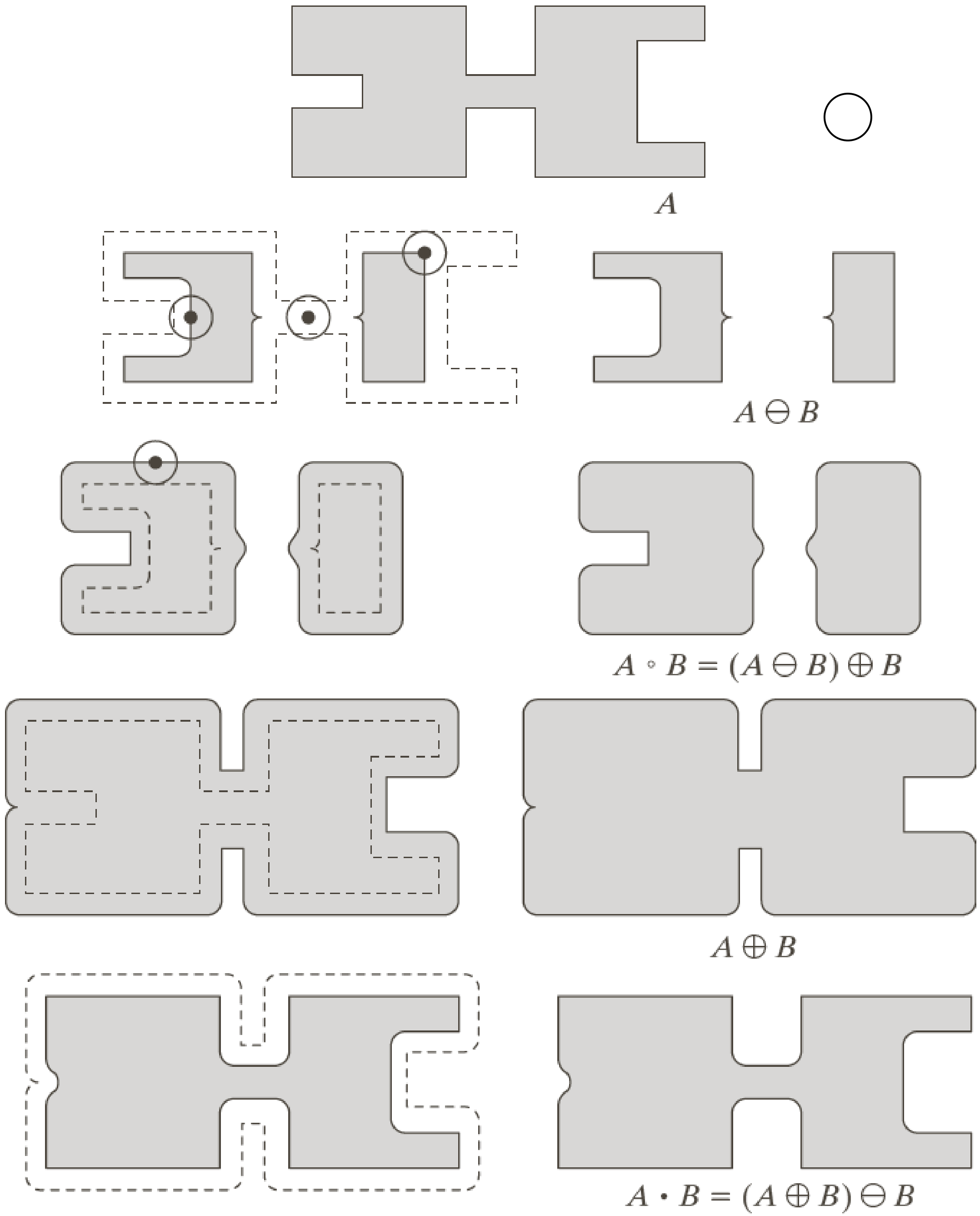
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Example: Closing



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.



	a
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Duality of Opening and Closing

- Opening and closing are duals of each other with respect to set complementation and reflection

$$\left(A \bullet B\right)^c = (A^c \circ \hat{B})$$

$$\left(A \circ B\right)^c = (A^c \bullet \hat{B})$$

a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

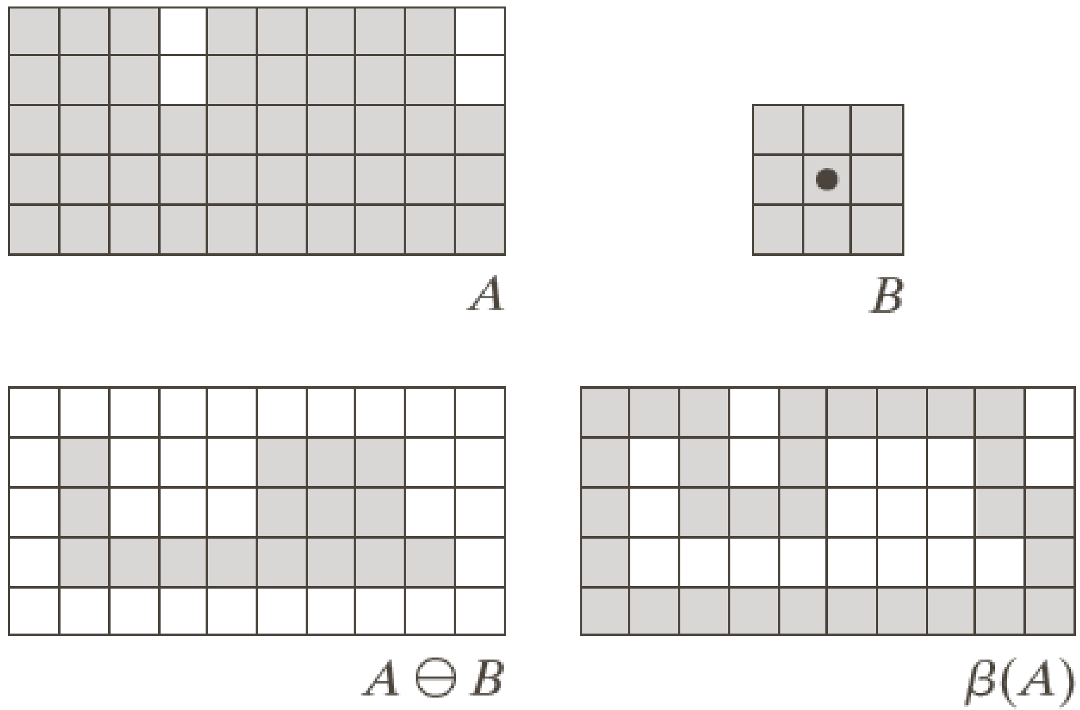
Some Basic Morphological Algorithms

- **Boundary Extraction**

  The boundary of a set A, can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - \left(A \ominus B\right)$$

Example 1



a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Example 2

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Some Basic Morphological Algorithms

- **Hole Filling**

  A hole may be defined as a background region surrounded by a connected border of foreground pixels.

  Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole).

  Given a point in each hole, the objective is to fill all the holes with 1s.

Some Basic Morphological Algorithms

► **Hole Filling**

1. Forming an array X_0 of 0s (the same size as the array containing A), except the locations in X_0 corresponding to the given point in each hole, which we set to 1.

2. $X_k = (X_{k-1} \oplus B) \cap A^c \quad k=1,2,3,\ldots$

Stop the iteration if $X_k = X_{k-1}$

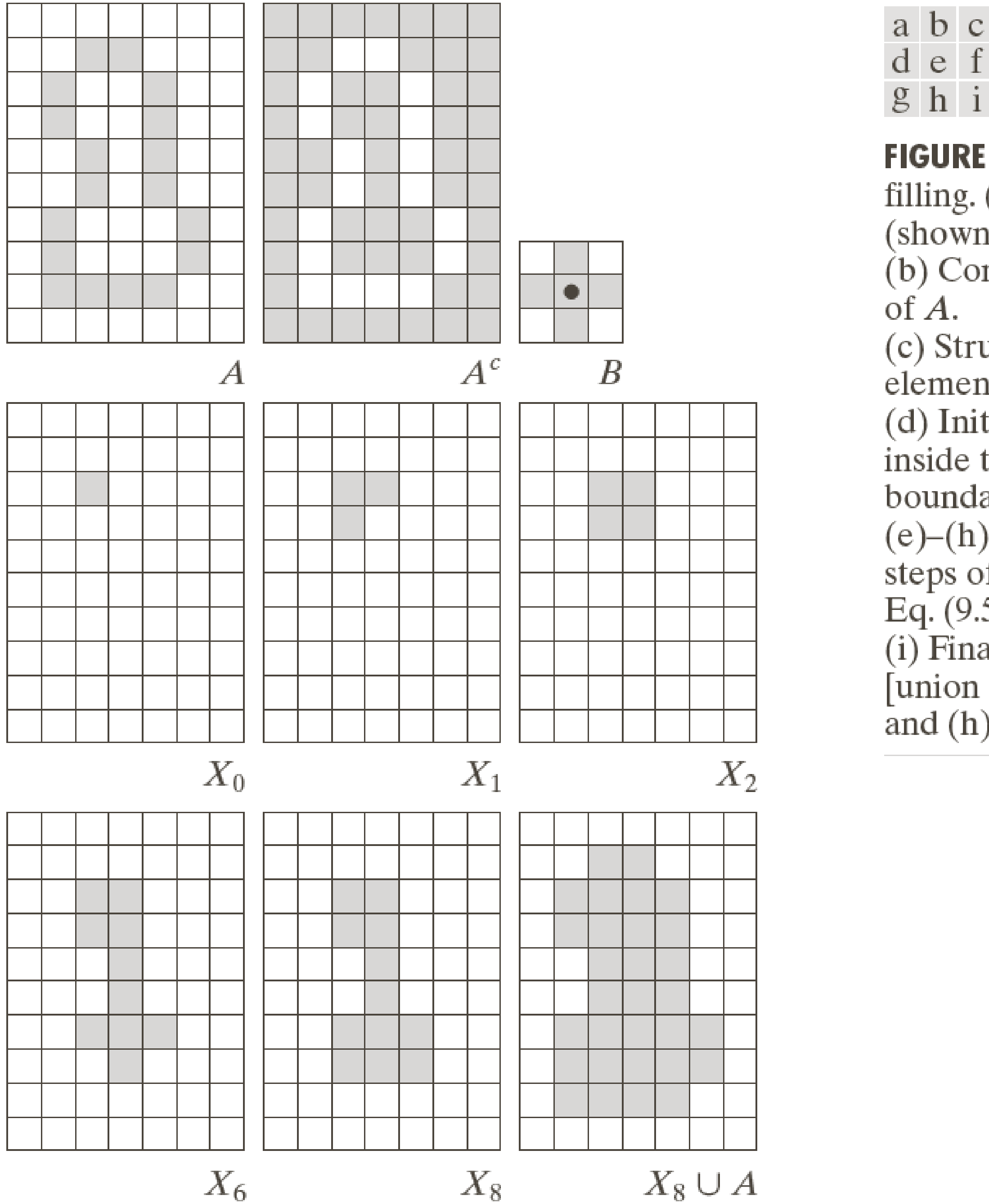
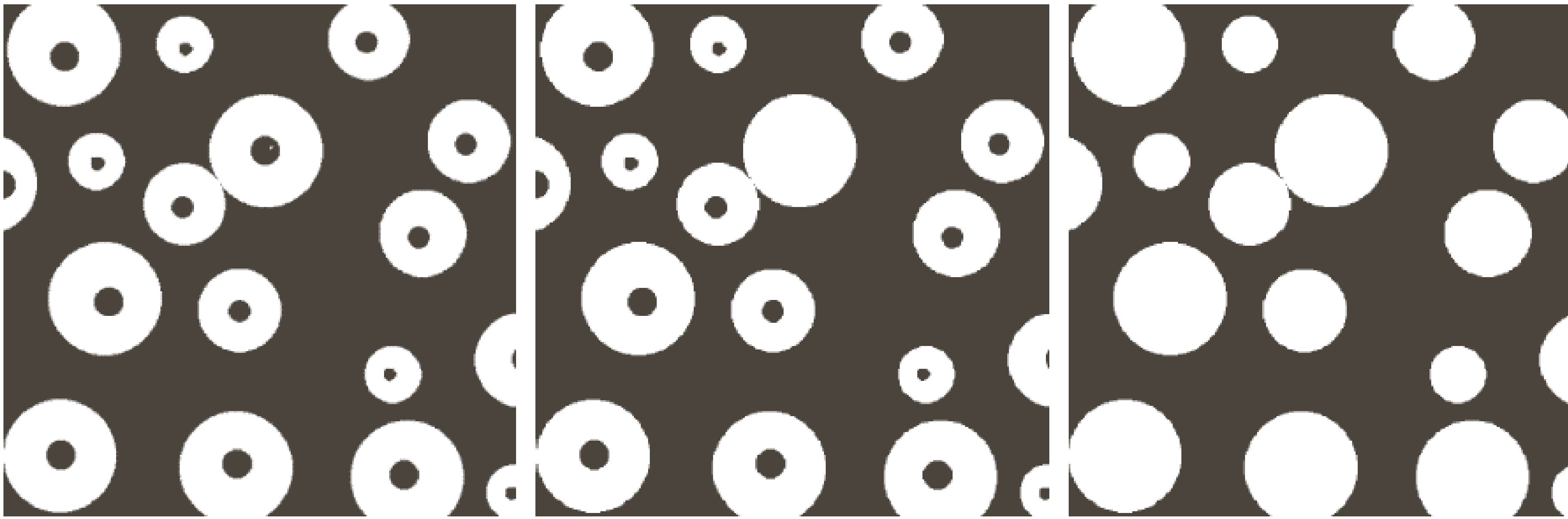


FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Some Basic Morphological Algorithms

- Thinning

  The thinning of a set A by a structuring element B, defined

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

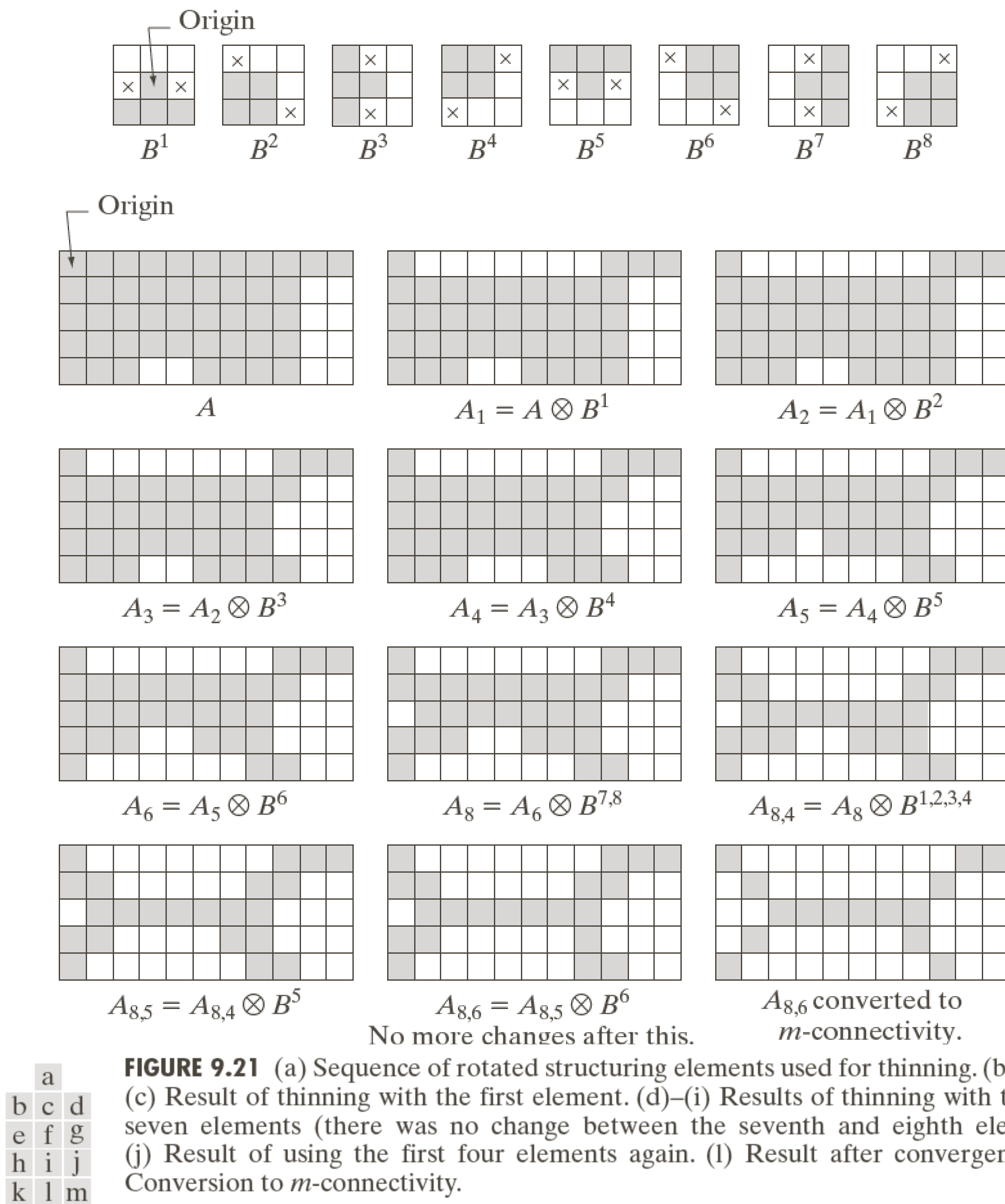


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

Some Basic Morphological Algorithms

- Thickening:

The thickening is defined by the expression

$$A \odot B = A \cup \left(A * B \right)$$

The thickening of A by a sequence of structuring element $\{B\}$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

Some Basic Morphological Algorithms

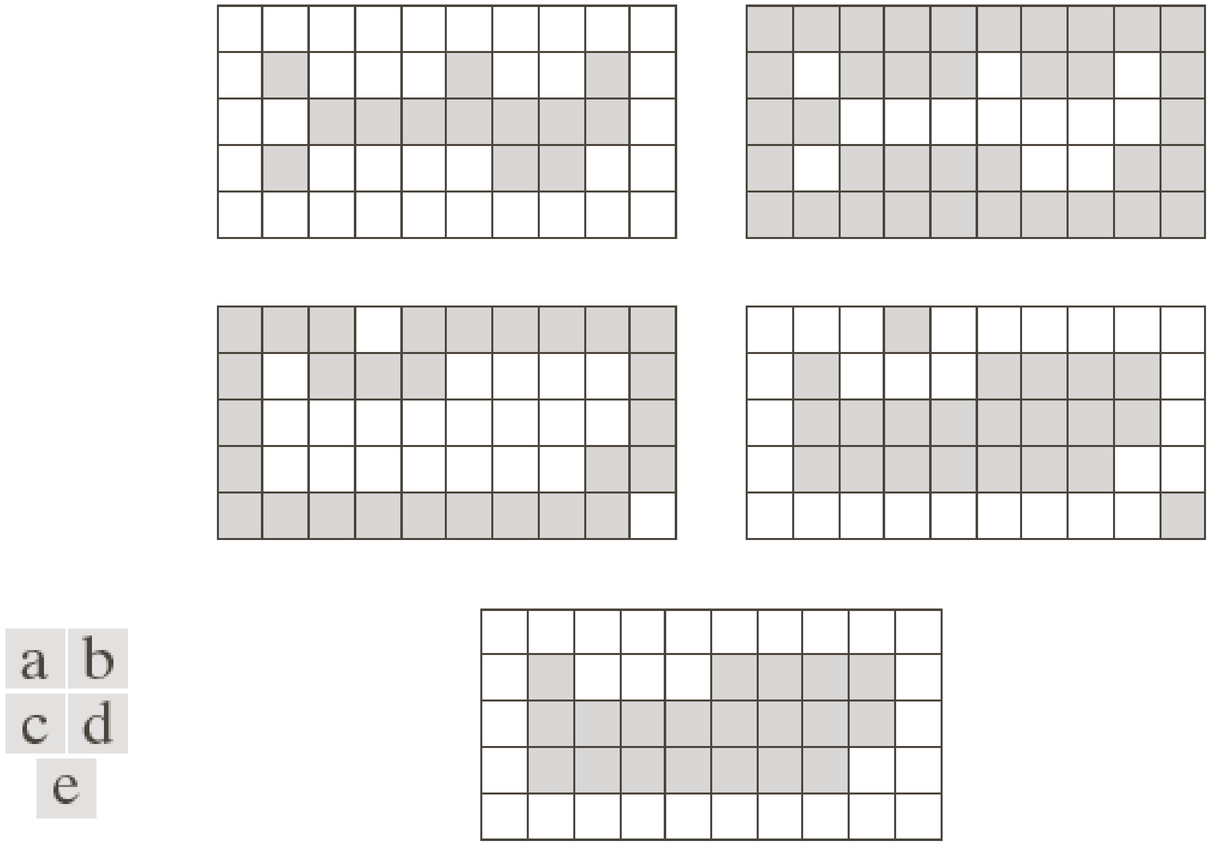


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Python Codes

- Erosion and Expansion

```
import cv2
import numpy as np

img = cv2.imread('input.png', 0)

kernel = np.ones((5,5), np.uint8)

img_erosion = cv2.erode(img, kernel, iterations=1)
img_dilation = cv2.dilate(img, kernel, iterations=1)

cv2.imshow('Input', img)
cv2.imshow('Erosion', img_erosion)
cv2.imshow('Dilation', img_dilation)

cv2.waitKey(0)
```

Python Codes

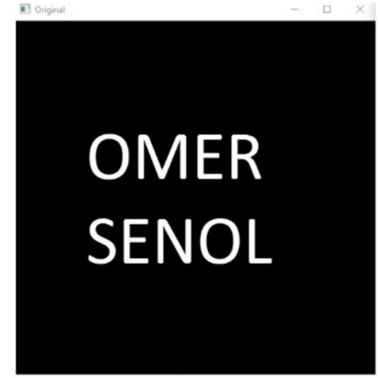
- Opening

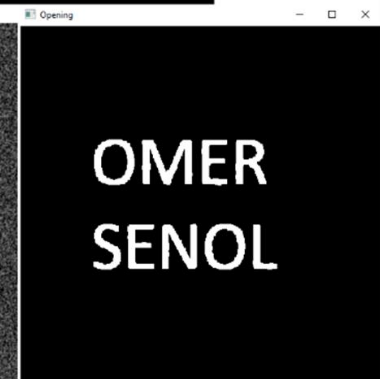
```python
img = cv2.imread("isim.png",0)
cv2.imshow("Original",img)
cv2.waitKey(0)

kernel = np.ones((5,5),dtype=np.uint8)

whiteNoise = np.random.randint(0,2,size=img.shape[:2])
whiteNoise = whiteNoise*255
noise_img = whiteNoise + img

opening = cv2.morphologyEx(noise_img.astype(np.float32),cv2.MORPH_OPEN,kernel)
cv2.imshow("Opening",opening)
cv2.waitKey(0)
```





Python Codes

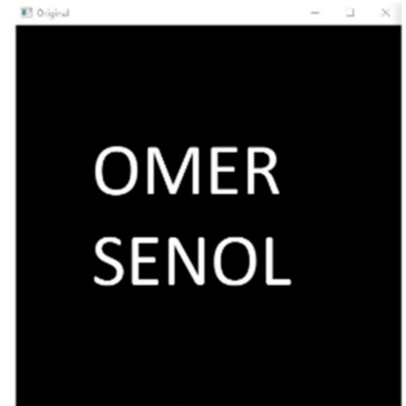
- Closing

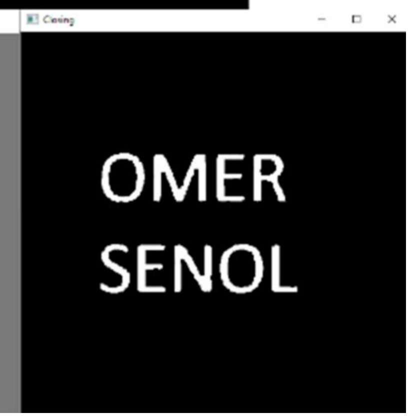
```python
img = cv2.imread("isim.png",0)
cv2.imshow("Original",img)
cv2.waitKey(0)

kernel = np.ones((5,5),dtype=np.uint8)

blackNoise = np.random.randint(0,2,size=img.shape[:2])
blackNoise = blackNoise*-255
noise_img = blackNoise + img
noise_img[noise_img <=-245] = 0

closing = cv2.morphologyEx(noise_img.astype(np.float32),cv2.MORPH_CLOSE,kernel)
cv2.imshow("Closing",closing)
cv2.waitKey(0)
```





Summary

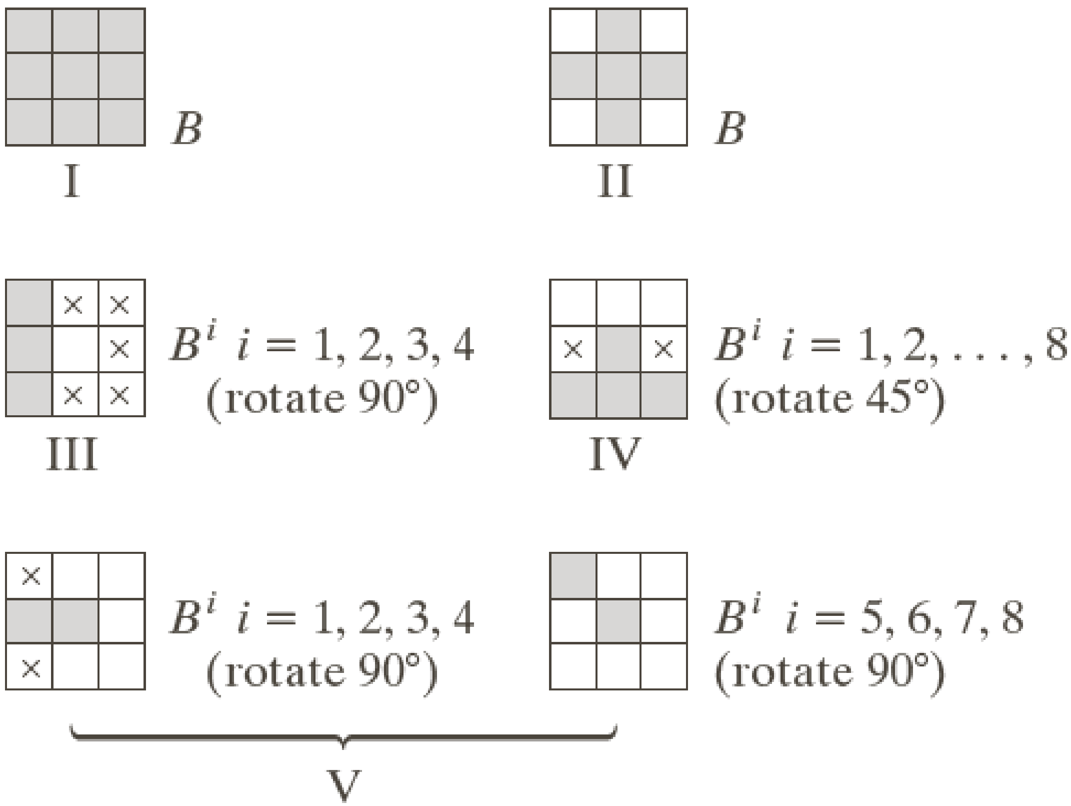


FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

Summary

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

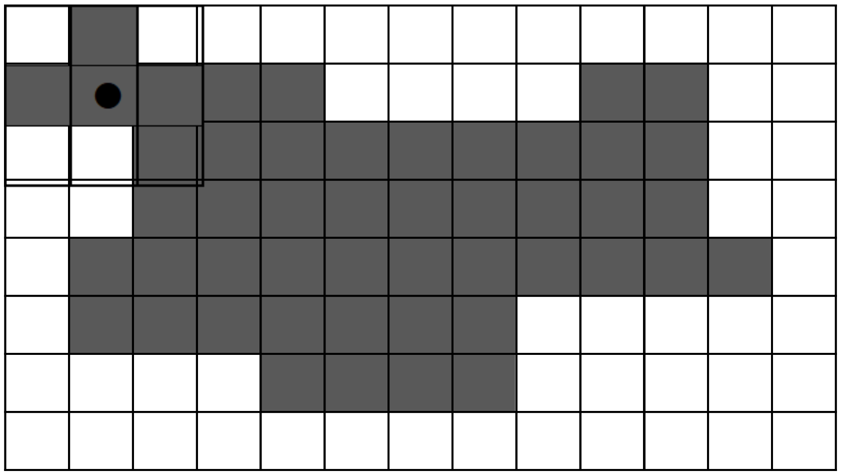
TABLE 9.1 Summary of morphological operations and their properties.

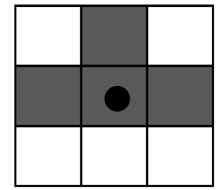
Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; $X_0 =$ array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; $X_0 =$ array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

(Continued)

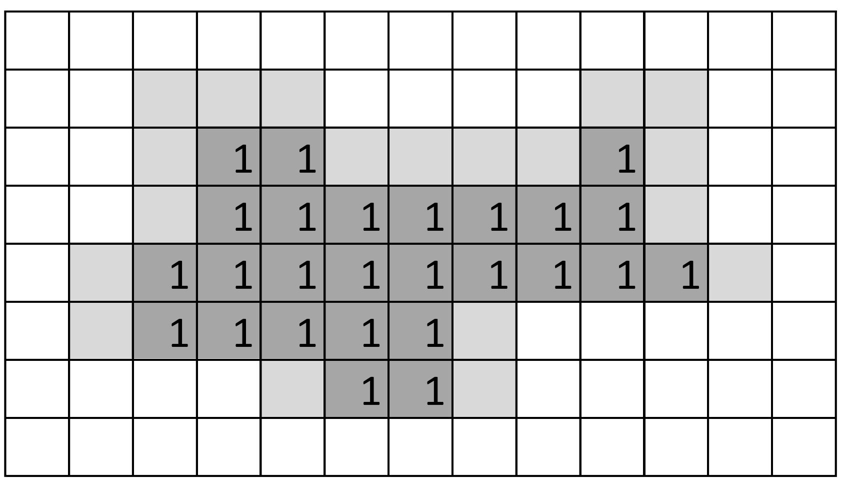
TABLE 9.1 *(Continued)*

Example Question: Erosion

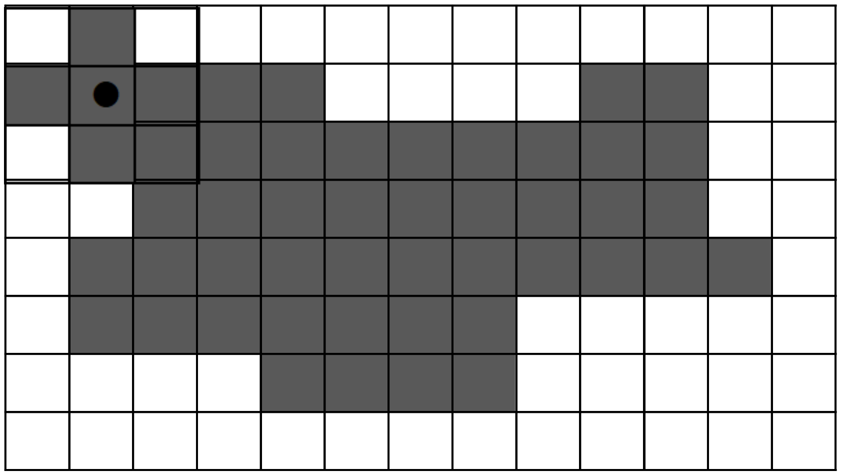


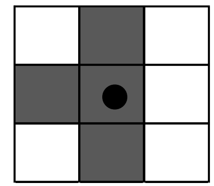


If perfect match center =1
If partial match center=0
No match center=0

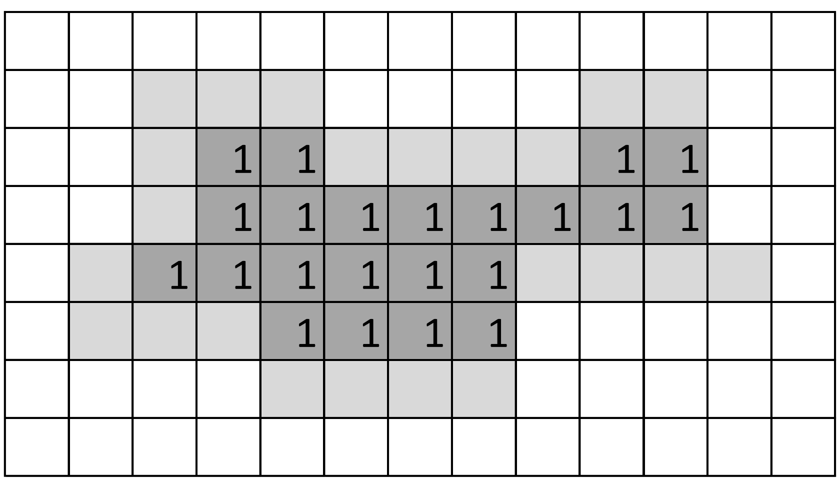


Example Question: Erosion

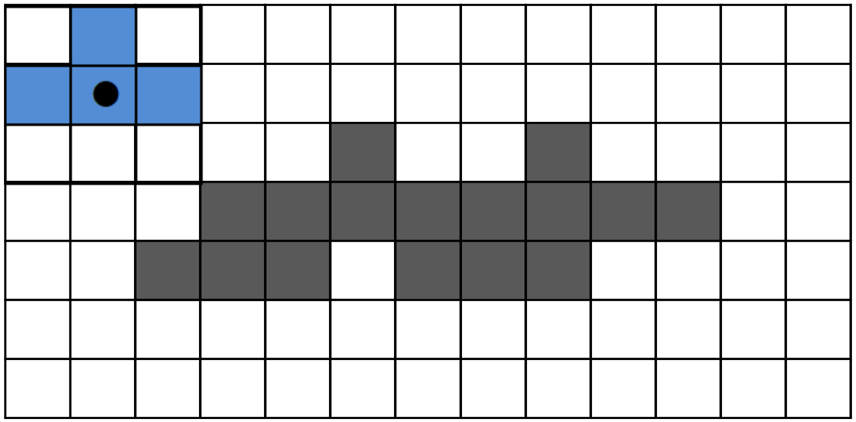


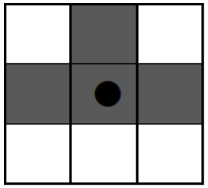


If perfect match center =1
If partial match center=0
No match center=0

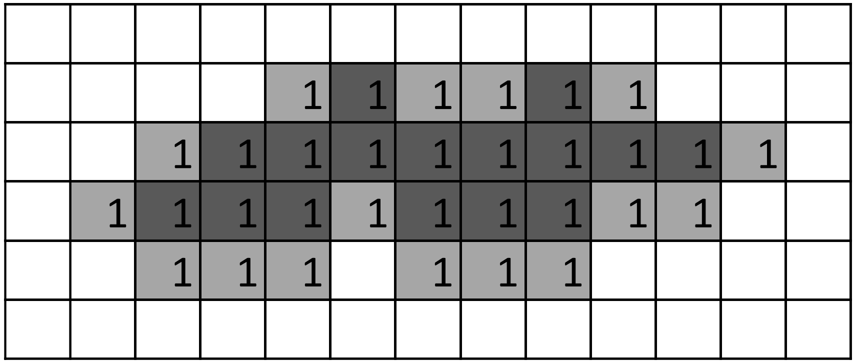


Example Question: Dilation

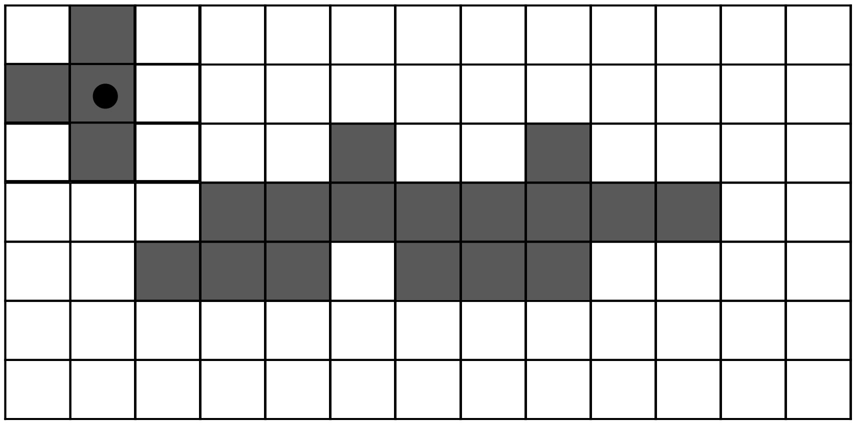


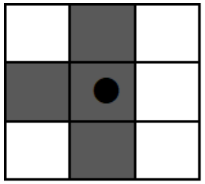


If perfect match center =1
If partial match center=1
No match center=0

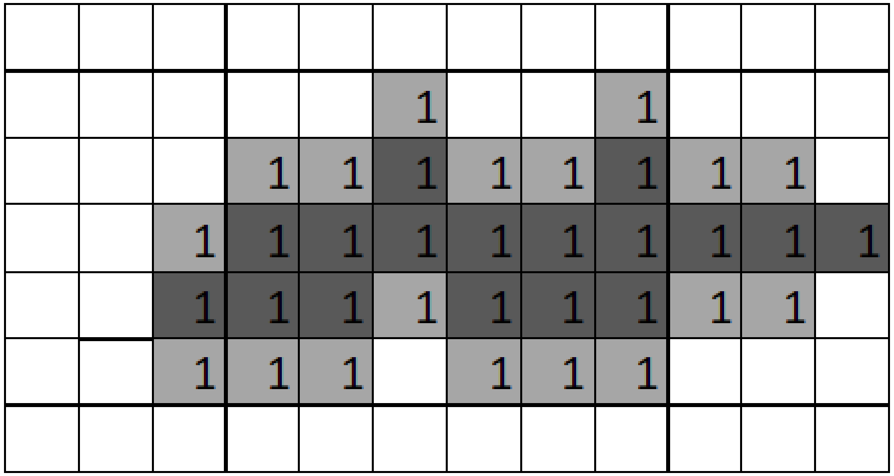


Example Question: Dilation





If perfect match	center =1
If partial match	center=1
No match	center=0



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