Part 5
Spatial Filtering

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If the facts don't fit the theory, change the facts.
~Einstein
Outline

3. Intensity Transformations and Spatial Filtering
   ► Some Basic Intensity Transformation Functions
   ► Histogram Processing
   ► Fundamentals of Spatial Filtering
   ► Smoothing Spatial Filters
   ► Sharpening Spatial Filters
   ► Combining Spatial Enhancement Methods
   ► Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering
Both zebras and dalmatians have black and white pixels in similar numbers.

The difference between the two is the characteristic appearance of small group of pixels rather than individual pixel values.
Spatial Filtering

A spatial filter consists of (a) a \textit{neighborhood}, and (b) a \textit{predefined operation}

Linear spatial filtering of an image of size MxN with a filter of size mxn is given by the expression

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$
FIGURE 3.28 The mechanics of linear spatial filtering using a $3 \times 3$ filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.
Spatial Filtering (Moving Average In 2D)
Spatial Filtering (Moving Average In 2D)

\[ F[x, y] \quad \text{and} \quad G[x, y] \]
Spatial Filtering (Moving Average In 2D)

\[ F[x, y] \]

\[ G[x, y] \]
Spatial Filtering (Moving Average In 2D)

\[ F[x, y] \quad G[x, y] \]
Spatial Filtering (Moving Average In 2D)

$F[x, y]$  
$G[x, y]$
Spatial Filtering (Moving Average In 2D)

\[
F[x, y] \\
G[x, y]
\]
Smoothing Spatial Filters

► Smoothing filters are used for blurring and for noise reduction.

► Blurring is used in removal of small details and bridging of small gaps in lines or curves.

► Smoothing spatial filters include linear filters and nonlinear filters.
Two Smoothing Averaging Filter Masks

Matlab Code:

\[
H = \text{ones(3)/9}; \\
\text{result} = \text{filter2}(H,\text{image});
\]

or

\[
\text{result} = \text{conv2}(	ext{image, ones(3)/9});
\]
Two Smoothing Averaging Filter Masks

clc
clear all
img=double(imread('cameraman.tif'));
imshow(uint8(img))
[m n]=size(img);
w = ones(3)
for i=0:m-3
    for j=0:n-3
        sum=0;
        for k=1:3
            for l=1:3
                sum = sum + img(i+k,j+l)*w(k,l);
            end
        end
        img1(i+1,j+1) = sum/9;
    end
end
img2 = uint8(img1);
figure
imshow(img2)
%imwrite(img2,‘output.png','png');
Two Smoothing Averaging Filter Masks

clc
clear all
img=(imread('cameraman.tif'));

imshow(img)
%img2=uint8(img);
w=(ones(3,3))/9;

%w(2,2)=1;

img2 = uint8(imfilter(img,w));
%imwrite(img2,'deneme_alcak_g.png','png');

figure
imshow(img2)
The general implementation for filtering an $M \times N$ image with a weighted averaging filter of size $m \times n$ is given

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t) \over \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)$$

where $m = 2a + 1$, $n = 2b + 1$. 

Spatial Smoothing Linear Filters
Spatial Smoothing Linear Filters

**FIGURE 3.33** (a) Original image, of size $500 \times 500$ pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15, 25, 35, 45,$ and $55$, respectively. The black squares at the top are of sizes $3, 5, 9, 15, 25, 35, 45,$ and $55$ pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size $50 \times 120$ pixels.
Example: Gross Representation of Objects

**FIGURE 3.34** (a) Image of size $528 \times 485$ pixels from the Hubble Space Telescope. (b) Image filtered with a $15 \times 15$ averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)
Order-statistic (Nonlinear) Filters

— Nonlinear

— Based on ordering (ranking) the pixels contained in the filter mask

— Replacing the value of the center pixel with the value determined by the ranking result

E.g., median filter, max filter, min filter

Matlab Code (median):
result = medfilt2(image);
Order-statistic (Nonlinear) Filters

clc
clear all
img=double(imread('glassware_noisy.png','png'));

[m n]=size(img);

p=3;
for i=1:m-p
  for j=1:n-p
    w=img(i:i+p-1,j:j+p-1);
    img2(i,j) = median(w(:));
  end
end
img2 = uint8(img2);
imshow(img2)
imwrite(img2,'median_filter_2015.png','png');
Example: Use of Median Filtering for Noise Reduction

**FIGURE 3.35** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a $3 \times 3$ averaging mask. (c) Noise reduction with a $3 \times 3$ median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)
Sharpening Spatial Filters

► The aim is to emphasize the transitions in intensity.

► Laplacian Operator

► Unsharp Masking and Highboost Filtering

► Using First-Order Derivatives for Nonlinear Image Sharpening — The Gradient
Sharpening Spatial Filters: Foundation

The first-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

The second-order derivative of f(x) as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$
FIGURE 3.36
Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.
The second-order isotropic derivative operator is the Laplacian for a function (image) $f(x,y)$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x + 1, y) + f(x - 1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y + 1) + f(x, y - 1) - 2f(x, y)$$

$$\nabla^2 f = f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1) - 4f(x, y)$$
Sharpening Spatial Filters: Laplace Operator

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6).
(b) Mask used to implement an extension of this equation that includes the diagonal terms.
(c) and (d) Two other implementations of the Laplacian found frequently in practice.
Image sharpening in the way of using the Laplacian:

\[ g(x, y) = f(x, y) + c \left[ \nabla^2 f(x, y) \right] \]

where,

- \( f(x, y) \) is input image,
- \( g(x, y) \) is sharpened images,
- \( c = -1 \) if \( \nabla^2 f(x, y) \) corresponding to Fig. 3.37(a) or (b) and \( c = 1 \) if either of the other two filters is used.
Sharpening Spatial Filters: Laplace Operator

**Matlab Code:**

```matlab
kernel = -1 * ones(3);
kernel(2,2) = 8;
% Now kernel = [-1,-1,-1; -1,8,-1; -1,-1,-1]
output = conv2(double(inputImage), kernel, 'same');
```

**Create predefined 2-D filter**

```matlab
input = imread('cameraman.tif');
h = fspecial('laplacian',alpha);
    alpha — Shape of the Laplacian 0.2 (default)
output = imfilter(input,h, 'replicate');
imshow(output);
```
Sharpening Spatial Filters: Laplace Operator

Create predefined 2-D filter

\[
\begin{align*}
    h &= \text{fspecial(type)} \\
    h &= \text{fspecial('average',hsize)} \\
    h &= \text{fspecial('disk',radius)} \\
    h &= \text{fspecial('gaussian',hsize,sigma)} \\
    h &= \text{fspecial('laplacian',alpha)} \\
    h &= \text{fspecial('log',hsize,sigma)} \\
    h &= \text{fspecial('motion',len,theta)} \\
    h &= \text{fspecial('prewitt')} \\
    h &= \text{fspecial('sobel')}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>'average'</td>
<td>Averaging filter</td>
</tr>
<tr>
<td>'disk'</td>
<td>Circular averaging filter (pillbox)</td>
</tr>
<tr>
<td>'gaussian'</td>
<td>Gaussian lowpass filter. Not recommended.</td>
</tr>
<tr>
<td>'laplacian'</td>
<td>Approximates the two-dimensional Laplacian</td>
</tr>
<tr>
<td>'log'</td>
<td>Laplacian of Gaussian filter</td>
</tr>
<tr>
<td>'motion'</td>
<td>Approximates the linear motion of a camera</td>
</tr>
<tr>
<td>'prewitt'</td>
<td>Prewitt horizontal edge-emphasizing filter</td>
</tr>
<tr>
<td>'sobel'</td>
<td>Sobel horizontal edge-emphasizing filter</td>
</tr>
</tbody>
</table>
FIGURE 3.38
(a) Blurred image of the North Pole of the moon.
(b) Laplacian without scaling.
(c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)
Image Sharpening based on First-Order Derivatives

For function $f(x, y)$, the gradient of $f$ at coordinates $(x, y)$ is defined as

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of vector $\nabla f$, denoted as $M(x, y)$

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$
Image Sharpening based on First-Order Derivatives

The magnitude of vector $\nabla f$, denoted as $M(x, y)$

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

$$M(x, y) \approx |g_x| + |g_y|$$

\[\begin{array}{ccc}
z_1 & z_2 & z_3 \\
z_4 & z_5 & z_6 \\
z_7 & z_8 & z_9 \end{array}\]

$$M(x, y) = |z_8 - z_5| + |z_6 - z_5|$$
Image Sharpening based on First-Order Derivatives

\[ \begin{array}{ccc} 
  z_1 & z_2 & z_3 \\
  z_4 & z_5 & z_6 \\
  z_7 & z_8 & z_9 \\
\end{array} \]

\[ \begin{array}{cc} 
  -1 & 0 \\
  0 & 1 \\
\end{array} \quad \begin{array}{cc} 
  0 & -1 \\
  1 & 0 \\
\end{array} \]

\[ \begin{array}{ccc} 
  -1 & -2 & -1 \\
  0 & 0 & 0 \\
  1 & 2 & 1 \\
\end{array} \quad \begin{array}{ccc} 
  -1 & 0 & 1 \\
  -2 & 0 & 2 \\
  -1 & 0 & 1 \\
\end{array} \]

**FIGURE 3.41**
A 3 × 3 region of an image (the zs are intensity values).
(b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.
Image Sharpening based on First-Order Derivatives

Roberts Cross-gradient Operators

\[ M(x, y) \approx |z_9 - z_5| + |z_8 - z_6| \]

Sobel Operators

\[ M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \]
\[ + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)| \]
Example

**FIGURE 3.42**
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o’clock).
(b) Sobel gradient.
(Original image courtesy of Pete Sites, Perceptics Corporation.)
Example:

Combining Spatial Enhancement Methods

Goal:

Enhance the image by sharpening it and by bringing out more of the skeletal detail.

**FIGURE 3.43**
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).
Example:

Combining Spatial Enhancement Methods

Goal:

Enhance the image by sharpening it and by bringing out more of the skeletal detail

FIGURE 3.43 (Continued)
(e) Sobel image smoothed with a $5 \times 5$ averaging filter. (f) Mask image formed by the product of (c) and (e).
(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)
References


► Lecture Notes, BIL717-Image Processing, Erkut Erdem

► Lecture Notes, EBM537-Image Processing, F.Karabiber